# LRS Bianchi Type-V Viscous Fluid Universe with a Time Dependent Cosmological Term Λ

Anirudh Pradhan [a,1], J. P. Shahi [b] and Chandra Bhan Singh [c]

[a] *Department of Mathematics, Hindu Post-graduate College, Zamania-232 331, Ghazipur, India*
*E-Addresses: pradhan@iucaa.ernet.in, acpradhan@yahoo.com*

[b,c] *Department of Mathematics, Harish Chandra Post-graduate College, Varanasi, India*

## Abstract

An LRS Bianchi type-V cosmological models representing a viscous fluid distribution with a time dependent cosmological term Λ is investigated. To get a determinate solution, the viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density. It turns out that the cosmological term $\Lambda(t)$ is a decreasing function of time, which is consistent with recent observations of type Ia supernovae. Various physical and kinematic features of these models have also been explored.

# 1    Introduction

Cosmological models representing the early stages of the Universe have been studied by several authors. An LRS (Locally Rotationally Symmetric) Binachi type-V spatially homogeneous space-time creates more interest due to its richer structure both physically and geometrically than the standard perfect fluid FRW models. An LRS Bianchi type-V universe is a simple generalization of the Robertson-Walker metric with negative curvature. Most cosmological models assume that the matter in the universe can be described by 'dust' (a pressureless distribution) or at best a perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of expanding universe [1]−[3]. It has been shown that bulk viscosity leads to inflationary like solution [4] and acts like a negative energy field in an expanding universe [5]. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of

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[1]Corresponding Author

radiation and matter during the recombination era. Bulk viscosity is associated with the Grand Unification Theories (GUT) phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [6] for a review on cosmological models with bulk viscosity). A number of authors have discussed cosmological solutions with bulk viscosity in various context [7]−[9].

Models with a relic cosmological constant $\Lambda$ have received considerable attention recently among researchers for various reasons (see Refs.[10]−[14] and references therein). Some of the recent discussions on the cosmological constant "problem" and consequence on cosmology with a time-varying cosmological constant by Ratra and Peebles [15], Dolgov [16]−[18] and Sahni and Starobinsky [19] have pointed out that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant". However, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying $\Lambda$ can be found. For these solutions, conservation of energy requires decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Earlier researchers on this topic, are contained in Zeldovich [20], Weinberg [11] and Carroll, Press and Turner [21]. Recent observations by Perlmutter et al. [22] and Riess et al. [23] strongly favour a significant and positive value of $\Lambda$. Their finding arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \leq z \leq 0.83$ and these suggest Friedmann models with negative pressure matter such as a cosmological constant ($\Lambda$), domain walls or cosmic strings (Vilenkin [24], Garnavich et al. [25]) Recently, Carmeli and Kuzmenko [26] have shown that the cosmological relativistic theory (Behar and Carmeli [27]) predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35} s^{-2}$. This value of "$\Lambda$" is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [25], Perlmutter et al. [22], Riess et al. [23], Schmidt et al. [28]). The main conclusion of these observations is that the expansion of the universe is accelerating.

Several ansätz have been proposed in which the $\Lambda$ term decays with time (see Refs. Gasperini [29, 30], Berman [31], Freese et al. [14], Özer and Taha [14], Peebles and Ratra [32], Chen and Hu [33], Abdussattar and Viswakarma [34], Gariel and Le Denmat [35], Pradhan et al. [36]). Of the special interest is the ansätz $\Lambda \propto S^{-2}$ (where $S$ is the scale factor of the Robertson-Walker metric) by Chen and Wu [33], which has been considered/modified by several authors ( Abdel-Rahaman [37], Carvalho et al. [14], Waga [38], Silveira and Waga [39], Vishwakarma [40]).

Recently Bali and Yadav [41] obtained an LRS Bianchi type-V viscous fluid cosmological models in general relativity. Motivated by the situations discussed above, in this paper, we focus upon the exact solutions of Einstein's field equa-

tions in presence of a bulk viscous fluid in an expanding universe. We do this by extending the work of Bali and Yadav [41] by including a time dependent cosmological term $\Lambda$ in the field equations. We have also assumed the coefficient of bulk viscosity to be a power function of mass density. This paper is organized as follows. The metric and the field equations are presented in section 2. In section 3 we deal with the solution of the field equations in presence of viscous fluid. The sections 3.1 and 3.2 contain the two different cases and also contain some physical aspects of these models respectively. Section 4 describe two models under suitable transformations. Finally in section 5 concluding remarks have been given.

## 2  The Metric and Field Euations

We consider LRS Bianchi type-V metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x}(dy^2 + dz^2), \tag{1}$$

where A and B are functions of $t$ alone.

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -8\pi T_i^j, \tag{2}$$

where $R_i^j$ is the Ricci tensor; $R = g^{ij}R_{ij}$ is the Ricci scalar; and $T_i^j$ is the stress energy-tensor in the presence of bulk stress given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - (v_{i;}^j + v_{;i}^j + v^j v^\ell v_{i;\ell} + v_i v^\ell v_{;\ell}^j)\eta$$

$$- \left(\xi - \frac{2}{3}\eta\right)v_{;\ell}^\ell(g_i^j + v_i v^j). \tag{3}$$

Here $\rho$, $p$, $\eta$ and $\xi$ are the energy density, isotropic pressure, coefficients of shear viscosity and bulk viscous coefficient respectively and $v^i$ the flow vector satisfying the relations

$$g_{ij}v^i v^j = -1. \tag{4}$$

The semicolon (; ) indicates covariant differentiation. We choose the coordinates to be comoving, so that $v^i = \delta_4^i$.

The Einstein's field equations (2) for the line element (1) has been set up as

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{A^2} = -8\pi\left[p - 2\eta\frac{A_4}{A} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - \Lambda, \tag{5}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = -8\pi\left[p - 2\eta\frac{B_4}{B} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - \Lambda, \tag{6}$$

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$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{3}{A^2} = -8\pi\rho - \Lambda, \tag{7}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0. \tag{8}$$

The suffix 4 after the symbols $A$, $B$ denotes ordinary differentiation with respect to $t$ and

$$\theta = v^\ell_{;\ell}$$

## 3   Solutions of the Field Eqations

In this section, we have revisited the solutions obtained by Bali and Yadav [41]. Equations (5) - (8) are four independent equations in seven unknowns $A$, $B$, $p$, $\rho$, $\xi$, $\eta$ and $\Lambda$. For complete determinacy of the system, we need three extra conditions.

Eq. (8), after integration, reduce to

$$A = B^k, \tag{9}$$

where $k$ is an integrating constant. Equations (5) and (6) lead to

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} - \frac{B_4^2}{B^2} - \frac{A_4 B_4}{AB} = -16\pi\eta\left(\frac{B_4}{B} - \frac{A_4}{A}\right). \tag{10}$$

Using Eq. (9) in (10), we obtain

$$\frac{df}{dB} + \left(\frac{k+1}{B}\right)f = -16\pi\eta, \tag{11}$$

where $B_4 = f(B)$. Eq. (11) leads to

$$f = -\frac{16\pi\eta}{(k+2)}B + \frac{L}{B^{k+1}}, \tag{12}$$

where $L$ is an integrating constant. Eq. (12) again leads to

$$B = (k+2)^{\frac{1}{k+2}}\left(k_1 - k_2 e^{-16\pi\eta t}\right)^{\frac{1}{k+2}}, \tag{13}$$

where

$$k_1 = \frac{L}{16\pi\eta}, \tag{14}$$

$$k_2 = \frac{N}{16\pi\eta}, \tag{15}$$

N being constant of integration. From Eqs. (9) and (13), we obtain

$$A = (k+2)^{\frac{k}{k+2}}\left(k_1 - k_2 e^{-16\pi\eta t}\right)^{\frac{k}{k+2}}. \tag{16}$$

Hence the metric (1) reduces to the form

$$ds^2 = -dt^2 + (k+2)^{\frac{2k}{k+2}} \left(k_1 - k_2 e^{-16\pi\eta t}\right)^{\frac{2k}{k+2}} dx^2$$

$$+ e^{2x}(k+2)^{\frac{2}{k+2}} \left(k_1 - k_2 e^{-16\pi\eta t}\right)^{\frac{2}{k+2}} (dy^2 + dz^2). \tag{17}$$

The pressure and density of the model (17) are obtained as

$$8\pi p = \frac{(8\pi)(16\pi\eta)k_2 e^{-16\pi\eta t}}{3(k+2)^2(k_1 - k_2 e^{-16\pi\eta t})^2} \Big[ k_1(k+2)^2(4\eta + 3\xi) - \{k^2(4\eta + 3\xi)$$

$$+ 4k(\eta + 3\xi) + 2(5\eta + 6\xi)\}k_2 e^{-16\pi\eta t}\Big] + \frac{1}{[(k+2)(k_1 - k_2 e^{-16\pi\eta t})]^{\frac{2k}{k+2}}} - \Lambda, \tag{18}$$

$$8\pi\rho = -\frac{(2k+1)}{(k+2)^2}(16\pi\eta)^2 k_2^2 \frac{e^{-32\pi\eta t}}{(k_1 - k_2 e^{-16\pi\eta t})^2}$$

$$+ \frac{3}{[(k+2)(k_1 - k_2 e^{-16\pi\eta t})]^{\frac{2k}{k+2}}} + \Lambda. \tag{19}$$

The expansion $\theta$ in the model (17) is obtained as

$$\theta = \frac{(16\pi\eta)k_2 e^{-16\pi\eta t}}{(k_1 - k_2 e^{-16\pi\eta t})}. \tag{20}$$

For complete determinacy of the system we have to consider three extra conditions. Firstly we assume that the coefficient of shear viscosity is constant, i.e., $\eta = \eta_0$ (say). For the specification of $\Lambda(t)$, we secondly assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{21}$$

where $\gamma(0 \leq \gamma \leq 1)$ is a constant.

Thirdly bulk viscosity $(\xi)$ is assumed to be a simple power function of the energy density [42]−[45].

$$\xi(t) = \xi_0 \rho^n, \tag{22}$$

where $\xi_0$ and $n$ are constants. For small density, $n$ may even be equal to unity as used in Murphy's work [46] for simplicity. If $n = 1$, Eq. (22) may correspond to a radiative fluid [47]. Near the big bang, $0 \leq n \leq \frac{1}{2}$ is a more appropriate assumption [48] to obtain realistic models.

For simplicity and realistic models of physical importance, we consider the following two cases $(n = 0, 1)$:

## 3.1   Model I:      Solution for $n = 0$

When $n = 0$, Eq. (22) reduces to $\xi = \xi_0 = $ constant. Hence, in this case Eqs. (18) and (19), with the use of (21), lead to

$$8\pi(1 + \gamma)\rho = \frac{8\pi M}{3N^2}\Big[k_1(k + 2)^2(4\eta_0 + 3\xi_0) - \{k^2(4\eta_0 + 3\xi_0)$$

$$+ 4k(\eta_0 + 3\xi_0) + 2(5\eta_0 + 6\xi_0)\}k_2 e^{-16\pi\eta_0 t}\Big] - \frac{(2k + 1)M^2}{N^2} + \frac{4}{N^{\frac{2k}{k+2}}}. \quad (23)$$

Eliminating $\rho(t)$ between Eqs. (19) and (23), we obtain

$$(1 + \gamma)\Lambda = \frac{8\pi M}{3N^2}\Big[k_1(k + 2)^2(4\eta_0 + 3\xi_0) - \{k^2(4\eta_0 + 3\xi_0)$$

$$+ 4k(\eta_0 + 3\xi_0) + 2(5\eta_0 + 6\xi_0)\}k_2 e^{-16\pi\eta_0 t}\Big] + (2k + 1)\gamma\frac{M^2}{N^2} + \frac{(1 - 3\gamma)}{N^{\frac{2k}{k+1}}}, \quad (24)$$

where

$$M = 16\pi k_2 \eta_0 e^{-16\pi\eta_0 t},$$

$$N = (k + 2)(k_1 - k_2 e^{-16\pi\eta_0 t}),$$

$$P = 2k^2 + 2k + 5,$$

$$Q = k^2 + 4k + 4. \quad (25)$$

## 3.2   Model II:      Solution for $n = 1$

When $n = 1$, Eq. (22) reduces to $\xi = \xi_0\rho$ . Hence, in this case Eqs. (18) and (19), with the use of (21), leads to

$$8\pi\rho = \frac{16\pi M\{2k_1(k + 2)^2\eta_0 - Pk_2\eta_0 e^{-16\pi\eta_0 t}\}}{3\left[(1 + \gamma)N^2 - M\{k_1(k + 2)^2\xi_0 - Qk_2\xi_0 e^{-16\pi\eta_0 t}\}\right]}$$

$$+ \frac{4N^{\frac{4}{k+2}} - (2k + 1)M^2}{\left[(1 + \gamma)N^2 - M\{k_1(k + 2)^2\xi_0 - Qk_2\xi_0 e^{-16\pi\eta_0 t}\}\right]}. \quad (26)$$

Eliminating $\rho(t)$ between Eqs. (19) and (26), we get

$$\Lambda = 16\pi M[2k_1(k + 2)^2\eta_0 - Pk_2\eta_0 e^{-16\pi\eta_0 t}] + \frac{\gamma(2k + 1)}{(1 + \gamma)}\frac{M^2}{N^2} + \frac{(1 - 3\gamma)}{(1 + \gamma)N^{\frac{2k}{k+2}}} +$$

$$\frac{M[k_1(k + 2)^2\xi_0 - Qk_2\xi_0 e^{-16\pi\eta_0 t}]\{4N^{\frac{4}{k+2}} - (2k + 1)M^2\}}{(1 + \gamma)N^2\left[(1 + \gamma)N^2 - M\{k_1(k + 2)^2\xi_0 - Qk_2\xi_0 e^{-16\pi\eta_0 t}\}\right]} \quad (27)$$

From Eqs. (23) and (26), we note that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all time in both models. The behaviour of the universe in these models will be determined by the cosmological term $\Lambda$; this term has the same effect as a uniform mass density $\rho_{eff} = -\Lambda/4\pi G$, which is constant in space

and time. A positive value of $\Lambda$ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of $\Lambda$, the expansion will tend to accelerate; whereas in the universe with negative value of $\Lambda$, the expansion will slow down, stop and reverse. From Eqs. (24) and (27), we observe that the cosmological term $\Lambda$ in both models is a decreasing function of time and it approaches a small positive value as time increase more and more. This is a good agreement with recent observations of supernovae Ia (Garnavich et al. [25], Perlmutter et al. [22], Riess et al. [23], Schmidt et al. [28]).

The shear $\sigma$ in the model (17) is given by

$$\sigma = \frac{(k-1)M}{\sqrt{3}N}. \tag{28}$$

The non-vanishing components of conformal curvature tensor are given by

$$C_{2323} = -C_{1414} = \frac{(k-1)M}{3N^2}[kM - 16\pi\eta_0 k_1(k+2)], \tag{29}$$

$$C_{1313} = -C_{2424} = \frac{(k-1)M}{3N^2}[16\pi\eta_0 k_1(k+2) - kM], \tag{30}$$

$$C_{1212} = -C_{3434} = \frac{(k-1)M}{3N^2}[16\pi\eta_0 k_1(k+2) - kM]. \tag{31}$$

Equations (20) and (28) lead to

$$\frac{\sigma}{\theta} = \frac{(k-1)}{\sqrt{3}(k+2)} = \text{constant}. \tag{32}$$

The model (17) is expanding, non-rotating and shearing. Since $\frac{\sigma}{\theta} = \text{conatant}$, hence the model does not approach isotropy. The space-time (17) is Petrov type D in presence of viscosity.

## 4    Other Models

After using the transformation

$$k_1 - k_2 e^{-16\pi\eta t} = \sin(16\pi\eta\tau), \quad k+2 = 1/16\pi\eta, \tag{33}$$

the metric (17) reduces to

$$ds^2 = -\left[\frac{\cos(16\pi\eta\tau)}{k_1 - \sin(16\pi\eta\tau)}\right]^2 d\tau^2 + \left[\frac{\sin(16\pi\eta\tau)}{16\pi\eta}\right]^{2(1-32\pi\eta)} dx^2$$

$$+ e^{2x}\left[\frac{\sin(16\pi\eta\tau)}{16\pi\eta}\right]^{(32\pi\eta)}(dy^2 + dz^2). \tag{34}$$

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The pressure $(p)$, density $(\rho)$ and the expansion $(\theta)$ of the model (34) are obtained as

$$8\pi p = \frac{(16\pi\eta)^2\{k_1 - \sin{(16\pi\eta\tau)}}{3\sin^2{(16\pi\eta\tau)}}\left[2k_1 - 2(1 - 48\pi\eta + 1152\pi^2\eta^2)\{k_1 - \sin{(16\pi\eta\tau)}\}\right]$$

$$+ \frac{(16\pi\eta)(8\pi\xi)\{k_1 - \sin{(16\pi\eta\tau)}\}}{\sin{(16\pi\eta\tau)}} + \left[\frac{16\pi\eta}{\sin{(16\pi\eta\tau)}}\right]^{2(1-32\pi\eta)} - \Lambda, \qquad (35)$$

$$8\pi\rho = \frac{2(24\pi\eta - 1)(16\pi\eta)^3\{k_1 - \sin{(16\pi\eta\tau)}\}^2}{\sin^2{(16\pi\eta\tau)}} + 3\left[\frac{16\pi\eta}{\sin{(16\pi\eta\tau)}}\right]^{2(1-32\pi\eta)} + \Lambda, \qquad (36)$$

$$\theta = \frac{(16\pi\eta)\{k_1 - \sin{(16\pi\eta\tau)}\}}{\sin{(16\pi\eta\tau)}}. \qquad (37)$$

## 4.1   Model I:       Solution for $n = 0$

When $n = 0$, Eq. (22) reduces to $\xi = \xi_0 = $ constant. Hence, in this case Eqs. (35) and (36), with the use of (21), lead to

$$8\pi(1 + \gamma)\rho = \frac{2(16\pi\eta_0)^2 M_1}{3\sin^2(16\pi\eta_0\tau)}\left[k_1 - P_1 M_1\right] + \frac{(16\pi\eta_0)(8\pi\xi_0)M_1}{\sin(16\pi\eta_0\tau)}$$

$$+ 4N_1 + \frac{2(24\pi\eta_0 - 1)(16\pi\eta_0)^3 M_1^2}{\sin^2(16\pi\eta_0\tau)}. \qquad (38)$$

Eliminating $\rho(t)$ between Eqs. (36) and (38), we obtain

$$(1 + \gamma)\Lambda = \frac{2(16\pi\eta_0)^2 M_1}{3\sin^2(16\pi\eta_0\tau)}\left[k_1 - P_1 M_1\right] + \frac{(16\pi\eta_0)(8\pi\xi_0)M_1}{\sin(16\pi\eta_0\tau)}$$

$$+ (1 - 3\gamma)N_1 + \frac{2\gamma(24\pi\eta_0 - 1)(16\pi\eta_0)^3 M_1^2}{\sin^2(16\pi\eta_0\tau)}. \qquad (39)$$

## 4.2   Model II:       Solution for $n = 1$

When $n = 1$, Eq. (22) reduces to $\xi = \xi_0\rho$ . Hence, in this case Eqs. (35) and (36), with the use of (21), lead to

$$8\pi\rho = \frac{2(16\pi\eta_0)^2 M_1[(k_1 - P_1 M_1) + 3(24\pi\eta_0 - 1)(16\pi\eta_0)M_1]}{3\sin(16\pi\eta_0\tau)[(1 + \gamma)\sin(16\pi\eta_0\tau) - 16\pi\eta_0\xi_0 M_1]}$$

$$+ \frac{4N_1 \sin(16\pi\eta_0\tau)}{[(1 + \gamma)\sin(16\pi\eta_0\tau) - 16\pi\eta_0\xi_0 M_1]}. \qquad (40)$$

Eliminating $\rho(t)$ between Eqs. (36) and (40), we obtain

$$\Lambda = \frac{2(16\pi\eta_0)^2 M_1(k_1 - P_1 M_1)}{3\sin(16\pi\eta_0\tau)[(1 + \gamma)\sin(16\pi\eta_0\tau) - 16\pi\eta_0\xi_0 M_1]}$$

$$+ N_1 \frac{[3(16\pi\eta_0\xi_0)M_1 + (1 - 3\gamma)\sin(16\pi\eta_0\tau)]}{[(1 + \gamma)\sin(16\pi\eta_0\tau) - 16\pi\eta_0\xi_0 M_1]} +$$

$$\frac{2(24\pi\eta_0 - 1)(16\pi\eta_0)^3 M_1^2[\gamma(1 + \gamma)\sin(16\pi\eta_0\tau) - (1 - \gamma)(16\pi\eta_0\xi_0)M_1]}{(1 + \gamma)\sin^2(16\pi\eta_0\tau)[(1 + \gamma)\sin(16\pi\eta_0\tau) - (16\pi\eta_0\xi_0)M_1]}, \quad (41)$$

where

$$M_1 = k_1 - \sin(16\pi\eta_0\tau),$$

$$N_1 = \left[\frac{16\pi\eta_0}{\sin(16\pi\eta_0\tau)}\right]^{2(1-32\pi\eta)},$$

$$P_1 = 1 - 48\pi\eta_0 + 1152\pi^2\eta_0^2. \quad (42)$$

The shear ($\sigma$) in the model (34) is obtained as

$$\sigma = \frac{(1 - 48\pi\eta_0)(16\pi\eta_0)[k_1 - \sin(16\pi\eta_0\tau)}{\sqrt{3}\sin(16\pi\eta_0\tau)}. \quad (43)$$

The models descibed in cases 4.1 and 4.2 preserve the same properties as in the cases of 3.1 and 3.2.

# 5   Conclusions

We have obtained a new class of LRS Bianchi type-V cosmological models of the universe in presence of a viscous fluid distribution with a time dependent cosmological term $\Lambda$. We have revisited the solutions obtained by Bali and Yadav [41] and obtained new solutions which also generalize their work.

The cosmological constant is a parameter describing the energy density of the vacuum (empty space), and a potentially important contribution to the dynamical history of the universe. The physical interpretation of the cosmological constant as vacuum energy is supported by the existence of the "zero point" energy predicted by quantum mechanics. In quantum mechanics, particle and antiparticle pairs are consistently being created out of the vacuum. Even though these particles exist for only a short amount of time before annihilating each other they do give the vacuum a non-zero potential energy. In general relativity, all forms of energy should gravitate, including the energy of vacuum, hence the cosmological constant. A negative cosmological constant adds to the attractive gravity of matter, therefore universes with a negative cosmological constant are invariably doomed to re-collapse [49]. A positive cosmological constant resists the attractive gravity of matter due to its negative pressure. For most universes, the positive cosmological constant eventually dominates over the attraction of matter and drives the universe to expand exponentially [50].

The cosmological constants in all models given in Sections 3.1 and 3.2 are decreasing functions of time and they all approach a small and positive value at late times which are supported by the results from recent type Ia supernova

observations recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich *et al.* [25], Perlmutter *et al.* [22], Riess *et al.* [23], Schmidt *et al.* [28]). Thus, with our approach, we obtain a physically relevant decay law for the cosmological term unlike other investigators where *adhoc* laws were used to arrive at a mathematical expressions for the decaying vacuum energy. Our derived models provide a good agreement with the observational results. We have derived value for the cosmological constant $\Lambda$ and attempted to formulate a physical interpretation for it.

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# References

[1] C. W. Misner, Astrophys. J. **151**, 431 (1968).

[2] G. F. R. Ellis, In *General Relativity and Cosmology*, Enrico Fermi Course, R. K. Sachs. ed. (Academic Press, New York, 1979).

[3] B. L. Hu, In *Advance in Astrophysics*, eds. L. J. Fung and R. Ruffini, (World Scientific, Singapore, 1983).

[4] T. Padmanabhan and S. M. Chitre, Phys. Lett. A **120**, 433 (1987).

[5] V. B. Johri and R. Sudarshan, *Proc. Int. Conf. on Mathematical Modelling in Science and Technology*, L. S. Srinath et al., eds (World Scientific, Singapore, 1989).

[6] Ø. Grøn, Astrophys. Space Sci. **173**, 191 (1990).

[7] A. Pradhan, V. K. Yadav and I. Chakrabarty, Int. J. Mod. Phys. D **10**, 339 (2001).
I. Chakrabarty, A. Pradhan and N. N. Saste, Int. J. Mod. Phys. D **10**, 741 (2001).
A. Pradhan and I. Aotemashi, Int. J. Mod. Phys. D **11**, 1419 (2002).
A. Pradhan and H. R. Pandey, Int. J. Mod. Phys. D **12** , 941 (2003).

[8] L. P. Chimento, A. S. Jakubi and D. Pavon, Class. Quant. Grav. **16**, 1625 (1999).

[9] G. P. Singh, S. G. Ghosh and A. Beesham, Aust. J. Phys. **50**, 903 (1997).

[10] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[11] S. Weinberg, *Gravitation and Cosmology*, (Wiley, New York, 1972).

[12] J. A. Frieman and I. Waga, Phys. Rev. D **57**, 4642 (1998).

[13] R. Carlberg, et al., Astrophys. J. **462**, 32 (1996).

[14] M. Özer and M. O. Taha, Nucl. Phys. B **287**, 776 (1987).
K. Freese, F. C. Adams, J. A. Frieman and E. Motta, ibid. B **287**, 1797 (1987).
J. C. Carvalho, J. A. S. Lima and I. Waga, Phys. Rev.D **46**, 2404 (1992).
V. Silviera and I. Waga, ibid. D **50**, 4890 (1994).

[15] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).

[16] A. D. Dolgov, in *The Very Early Universe*, eds. G. W. Gibbons, S. W. Hawking and S. T. C. Siklos, (Cambridge Univerity Press, 1983).

[17] A. D. Dolgov, M. V. Sazhin and Ya. B. Zeldovich, *Basics of Modern Cosmology*, (Editions Frontiers, 1990).

[18] A. D. Dolgov, Phys. Rev. D **55**, 5881 (1997).

[19] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000).

[20] Ya. B. Zeldovich, Sov. Phys.-Uspekhi **11**, 381 (1968).

[21] S. M. Carroll, W. H. Press and E. L. Turner, Ann. Rev. Astron. Astrophys. **30**, 499 (1992).

[22] S. Perlmutter et al., Astrophys. J. **483**, 565 (1997), Supernova Cosmology Project Collaboration (astro-ph/9608192);
S. Perlmutter et al., Nature **391**, 51 (1998), Supernova Cosmology Project Collaboration (astro-ph/9712212);
S. Perlmutter et al., Astrophys. J. **517**, 565 (1999), Project Collaboration (astro-ph/9608192).

[23] A. G. Riess et al., Astron. J. **116**, 1009 (1998); Hi-Z Supernova Team Collaboration (astro-ph/9805201).

[24] A. Vilenkin, Phys. Rep. **121**, 265 (1985).

[25] P. M. Garnavich et al., Astrophys. J. **493**, L53 (1998a), Hi-z Supernova Team Collaboration (astro-ph/9710123);
P. M. Garnavich et al., Astrophys. J. **509**, 74 (1998b); Hi-z Supernova Team Collaboration (astro-ph/9806396).

[26] M. Carmeli and T. Kuzmenko, Int. J. Theor. Phys. **41**, 131 (2002).

[27] S. Behar and M. Carmeli, Int. J. Theor. Phys. **39**, 1375 (2002) 1375.

[28] B. P. Schmidt et al., Astrophys. J. **507**, 46 (1998), Hi-z Supernova Team Collaboration (astro-ph/9805200).

[29] M. Gasperini, Phys. Lett. B **194**, 347 (1987).

[30] M. Gasperini, Class. Quant. Grav. **5**, 521 (1988).

[31] M. S. Berman, Int. J. Theor. Phys. **29**, 567 (1990) 567;
M. S. Berman, Int. J. Theor. Phys. **29**, 1419 (1990);
M. S. Berman, Phys. Rev. D **43**, 75 (1991).
M. S. Berman and M. M. Som, Int. J. Theor. Phys. **29**, 1411 (1990).
M. S. Berman, M. M. Som and F. M. Gomide, Gen. Rel. Grav. **21**, 287 (1989).
M. S. Berman and F. M. Gomide, Gen. Rel. Grav. **22**, 625 (1990).

[32] P. J. E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988).

[33] W. Chen and Y. S. Wu, Phys. Rev. D **41**, 695 (1990).

[34] Abdussattar and R. G. Vishwakarma, Pramana J. Phys. **47**, 41 (1996).

[35] J. Gariel and G. Le Denmat, Class. Quant. Grav. **16**, 149 (1999).

[36] A. Pradhan and A. Kumar, Int. J. Mod. Phys. D **10**, 291 (2001).
A. Pradhan and V. K. Yadav, Int J Mod Phys D **11**, 983 (2002).
A. Pradhan and O. P. Pandey, Int. J. Mod. Phys. D **12**, 941 (2003).
A. Pradhan, S. K. Srivastava and K. R. Jotania, Czech. J. Phys. **54**, 255 (2004).
A. Pradhan, A. K. Yadav and L. Yadav, Czech. J. Phys. **55**, 503 (2005).
A. Pradhan and P. Pandey, Czech. J. Phys. **55**, 749 (2005).
A. Pradhan and P. Pandey, Astrophys. Space Sci. **301**, 221 (2006).
G. S. Khadekar, A. Pradhan and M. R. Molaei, Int. J. Mod. Phys. D **15**, 95 (2006).
A. Pradhan, K. Srivastava and R. P. Singh, Fizika B (Zagreb) **15**, 141 (2006).
C. P. Singh, S. Kumar and A. Pradhan, Class. Quantum Grav. **24**, 455 (2007).
A. Pradhan, A. K. Singh and S. Otarod, Romanian J. Phys. **52**, 415 (2007).

[37] A.-M. M. Abdel-Rahaman, Gen. Rel. Grav. **22**, 655 (1990); Phys. Rev. D **45**, 3492 (1992).

[38] I. Waga, Astrophys. J. **414**, 436 (1993).

[39] V. Silveira and I. Waga, Phys. Rev. D **50**, 4890 (1994).

[40] R. G. Vishwakarma, Class. Quant. Grav. **17**, 3833 (2000).

[41] R. Bali and M. K. Yadav, J. Raj. Acad. Phys. Sci. **1**, 47 (2002).

[42] D. Pavon, J. Bafaluy and D. Jou, Class Quant. Grav. **8**, 357 (1991); "Proc. Hanno Rund Conf. on Relativity and Thermodynamics", Ed. S. D. Maharaj, (University of Natal, Durban, 1996, p. 21).

[43] R. Maartens, Class Quant. Grav. **12**, 1455 (1995).

[44] W. Zimdahl, Phys. Rev. D **53**, 5483 (1996).

[45] N. O. Santos, R. S. Dias and A. Banerjee, J. Math. Phys. **26**, 878 (1985).

[46] G. L. Murphy, Phys. Rev. **D 8**, 4231 (1973).

[47] S. Weinberg, Astrophys. J. **168**, 175 (1971).

[48] U. A. Belinskii and I. M. Khalatnikov, Sov. Phys. JETP **42**, 205 (1976).

[49] S. M. Carrol, W. H. Press and E. L. Turner, ARA&A **30**, 499 (1992).

[50] C. S. Kochanek, Astrophys. J. **384**, 1 (1992).